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REFINEMENT OF NON-LINEAR QUANTITATIVE ESTIMATES OF THE SAATI SCALE ADVANTAGES INDICATORS

Abstract. *Any effective management is accompanied by a solution multi-criteria decision-making problem (MCDMP). Among the relevant methods and technologies, the Analytic Hierarchy Process has been the leading efficiency for several decades (AHP), developed by T. L. Saaty. The implementation of AHP consists in determining local vectors of priorities — normalized weight coefficients (NWC) — at all levels of the hierarchy of the MCDMP being solved. Their further synthesis leads to a global vector — integrative assessments of the studied alternatives, which and only which have the systemic property of emergence, which facilitates a thorough choice of the best of them.*

Despite the positive results of numerous studies on the application and improvement of AHP, the need to construct a pairwise comparison matrix (PCM) remains unchanged, the solution of which actually leads to obtaining the desired priorities-NWC. The specified PCM is built based on direct and inverse linear score estimates-quantitative equivalents of linguistic inducers of preference (IP). Which is actually an artificial replacement of measurements in the scoring ranking scale with measurements in the interval scale. Therefore, the corresponding solutions are not optimal, because linearity is not a property of human thinking and orientation on it can lead to erroneous results.

The ranking of linguistic IRs of the Saaty scale is obvious, therefore, using the mathematical method of prioritization (MoP), known in systems analysis as the “leader problem”, nonlinear NWCs of all nine IPs, measured already in an absolute scale unique in qualimetric properties, were obtained. The acceptability for research of the results of the second iteration of the MoP is justified. The analytical description of the indicated nonlinearity was carried out in a step scale. It was established that the nonlinearity itself increased the sensitivity of the scale to the measurement of priority by 16.5 times. Quantitative estimates of the ratio of linguistic IPs associated with the modifier “very” improved in relation to the indicators of the linear scale: by 20 % for the ratio IP7–IP6, and by 17 % and 39 % for the ratio IP8–IP7 and IP8–IP6, respectively. This indicates a greater harmonization of the quantitative and qualitative adequacy of the Saaty’s scale.

Keywords: *decision making, analytic hierarchy process, Saaty’s scale, non-linearity dephasification of llinguistic indicators of preference, normalized weight coefficient.*

INTRODUCTION

Nowadays, the solution of MCDMP is an objective reality and an urgent need in any management. After all, the more different criteria and indicators that characterize the studied problem situation (PS) are used, the more comprehensively, and therefore effectively, this PS is solved. Moreover, with the use of a wide range of methods, namely multi-criteria optimization [1–5 and others].

Thus, research and development of MCDMP technology is a relevant scientific and practical problem.

STATEMENT OF THE PROBLEM

Most modern methods, technologies, and procedures for solving and optimizing MCDMPs (**Fig. 1**) are implemented in a special software environment [1–9, etc.], which makes them unusually attractive to users. Among these methods and technologies, AHP occupies a special place, which has been successfully used for a long time to solve MCDMPs in various fields of human activity: medicine, management, including government, education, etc.

AHP and its logical development — the method of network analysis (Analytic Network Processes, ANP), proposed by the same T. L. Saaty, have gained worldwide recognition: today there are about 17,000 (!) scientific papers devoted to the development of their practical application, specialized journals are published, and an international congress on the relevant topic is regularly held.

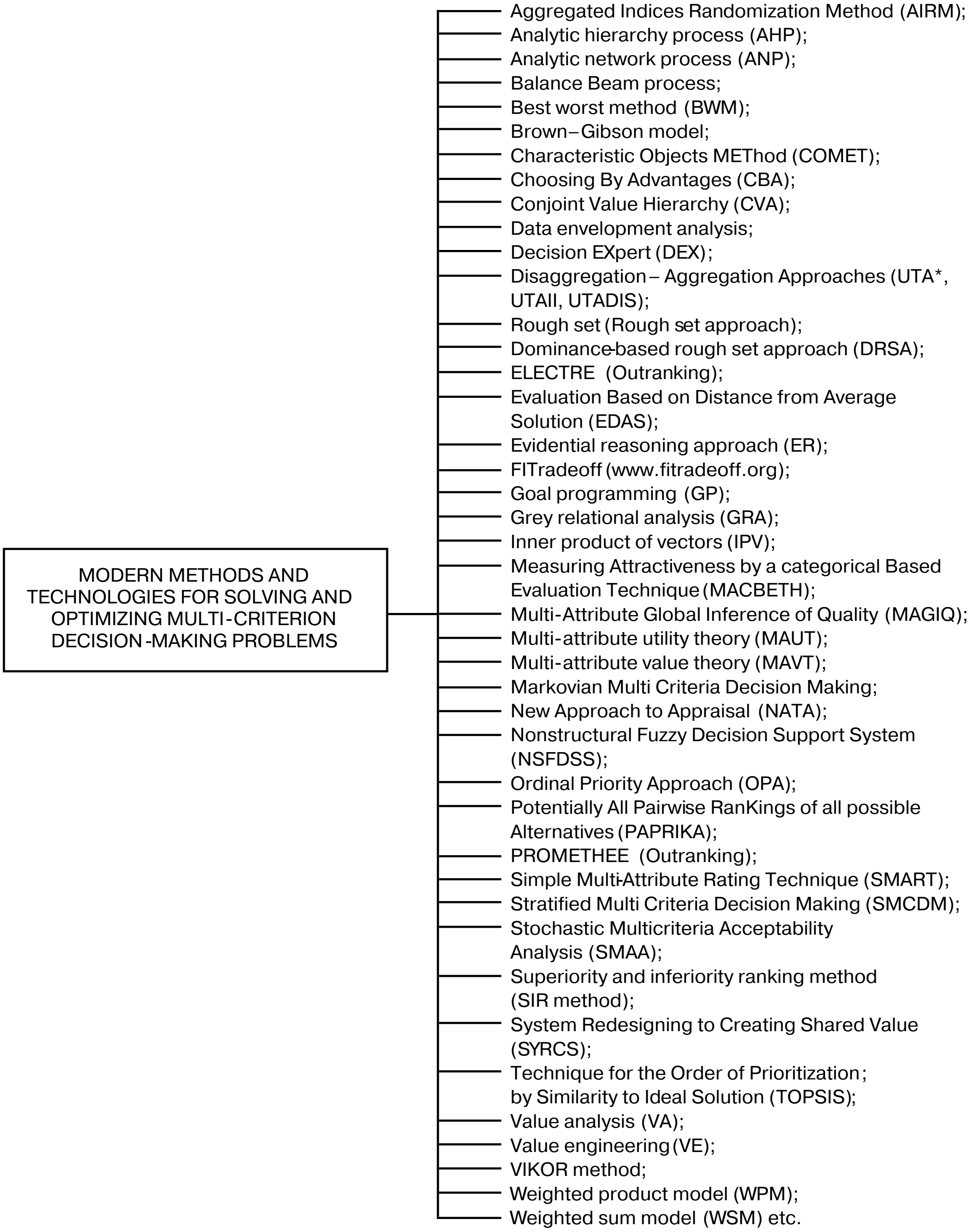

Fig. 1. Methods and technologies for solving multi-criteria decision-making problems

Thus, improving AHP/ANP is a valid way to develop the theory and practice of MCDMP.

ANALYSIS OF LATEST RESEARCH AND PUBLICATIONS

Note that all publications devoted to the application/optimization of AHP/ANPs are focused on the construction of PCM, $A=\|a_{ij}\|$ the elements of which are direct and inverse quantitative assessments of linguistic IPs of the corresponding Saati scale [10–12, etc.]:

$$A=\begin{Vmatrix} a_{11} & a_{12} & \dots & a_{1i} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2i} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{i1} & a_{i2} & \dots & a_{ii} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{ni} & \dots & a_{nn} \end{Vmatrix}, \qquad (1)$$

where $$a_{ij} = \begin{cases} LP_{IP_k}, & if \quad a_i \succ a_j \\ \dfrac{1}{LP_{IP_k}}, & if \quad a_j \succ a_i \\ 1, & if \quad i = j \end{cases}, \quad (2)$$

where LP_{IP_k} — linguistic IP quantitative assessment mark according to the Saati scale (**Table 1**).

Table 1

Quantitative assessments of linguistic indicators Saaty's scale

IP_k	Linguistic indicators of preference	Quantitative assessments of priority levels
1	2	3
IP_9	Extreme importance	$IP_{I_9} = 9$
IP_8	Very, very strong	$IP_{I_8} = 8$
IP_7	Very strong or demonstrated importance	$IP_{I_7} = 7$
IP_6	Strong plus	$IP_{I_6} = 6$
IP_5	Strong importance	$IP_{I_5} = 5$
IP_4	Moderate plus	$IP_{I_4} = 4$
IP_3	Moderate importance	$IP_{I_3} = 3$
IP_2	Weak	$IP_{I_2} = 2$
IP_1	Equal importance	$IP_{I_1} = 1$

Solving the PCM contributes to obtaining local and general vector priorities, and therefore the solution of the MCDMP. However, despite the positive results of applying the AHP/ANP, they are not optimal.

For example, the linguistic description uses the modifier "Very" ("Very strong or demonstrated importance"). Therefore, an adequate transition $IP_6 \rightarrow IP_7$ should occur only under the conditions of "concentration", according to the postulates of fuzzy mathematics, in the expert's imagination of the specified priority in the previous IP_6. The same applies to semantic understanding IP_8 ("Very, very strong preference"), which should take into account the specified "concentration" in both, IP_7 and IP_6.

The above can be formally presented as follows:

$$I_7 = CON(I_6) \Leftrightarrow \mu_{I_7}(\cdot) = \mu^2_{I_6}(\cdot), \quad (3)$$

$$I_8 = CON(I_7) \Leftrightarrow \mu_{I_8}(\cdot) = \mu^2_{I_7}(\cdot) \Leftrightarrow \Leftrightarrow \mu_{I_8}(\cdot) = \mu^4_{I_6}(\cdot). \quad (4)$$

Expressions (3), (4) convincingly indicate the sig-nificant nonlinearity of the linguistic scale of the IP, which is presented in column 2 of **Table 1**. Moreover, with a clearly pronounced linguistic transformation of some IPs. Which, in our opinion, should also be illus-trated by certain quantitative relationships. However, from column 3 of the same table we see that quan-titatively we are talking only about linear LP scores. That in order to ensure the possibility of carrying out certain mathematical transformations, the actual mea-surements in the qualitative ranking (ordering) score scale are artificially interpreted as measurements in the quantitative interval scale. The result is the actual quantitative assumption of the expert's li-near thinking, which determines the degree of priority of the compared alternatives, objects, phenomena, etc.

Note that linear thinking determines the expert's response to the priority of one alternative over another, directly proportional to the strength of this priority. Which clearly contradicts the content and semantic incidences of linguistic assessments of IR (column 2 of **Table 1**). On the other hand, linear assessments in a certain way coarsen the LP measurements.

In light of the above, let us consider the ratio of quantitative estimates of LP, focusing on expressions (3), (4) and column 3 of **Table 1**:

$$\frac{LP_{IP_7}}{LP_{IP_6}} = \frac{7}{6} = 1.17; \quad (5)$$

$$\begin{cases} \dfrac{LP_{IP_8}}{LP_{IP_7}} = \dfrac{8}{7} = 1.14 \\ \dfrac{LP_{IP_8}}{LP_{IP_6}} = \dfrac{8}{6} = 1.33 \end{cases}. \quad (6)$$

As we can see, the results (5), (6) clearly do not quantitatively correspond to the idea of the "con-centration" of priority, which was discussed above. Namely. From the ratios $IP_7 - IP_6$, $IP_8 - IP_7$, connec-ted by the fuzzy operation "concentration", the excess over the "basic" IPs (respectively IP_6 and IP_7) is established by only 17 % and 14 %. At the same time, for the ratio $IP_8 - IP_6$, when we are talking about a double "concentration" of priority, this excess is only 33 %.

It is also easy to find out (column 3 of **Table 1**) that the quantitative assessment of the sensitivity of priority measurements proposed by T. L. Saaty, as the ratio of their maximum and minimum levels LP_{IP_k}, is limited and equal to:

$$max\left(\frac{LP_{IP_k}}{LP_{IP_j}}\right) = \frac{max\ LP_{IP_k}}{min\ LP_{IP_j}} = \frac{LP_{IP_9}}{LP_{IP_1}} = \frac{9}{1} = 9. \quad (7)$$

In conclusion of this part of our article, we would like to point out that none of the publications from their huge spectrum devoted to the application/development of AHR and ANP consider the issue of improving the technology of constructing PCM by using nonlinear LP estimates. Although such a need is obvious.

PROBLEM STATEMENT

It seems possible to eliminate the shortcomings of the linear LP assessment scale (column 3 of **Table 1**) by providing each IP with a corresponding nonlinear NWC:

$$IP_k \Rightarrow \beta_{IP_k}: \quad 0 \le \beta_{IP_k} \le 1, \quad \sum_{k=1}^{n=9} \beta_{IP_k} = 1. \qquad (8)$$

Then the construction of the PCM should be based on such a transformation of expression (2).

$$a_{ij} = \begin{cases} \beta_{IP_k}, & if \quad a_i \succ a_j \\ \dfrac{1}{\beta_{IP_k}}, & if \quad a_j \succ a_i \\ 1, & if \quad i = j \end{cases} \qquad (9)$$

We consider it appropriate to use the MoP for this [11–13, etc.], which is more acceptable for solving this type of problem.

Based on the above, the purpose of this publication is to determine the NWC of Saaty's scale of IP, using the MoP and presenting it as a holistic 9-point scale. This assumption is somewhat different from the opinion of this respected scientist, who considers I_9, I_7, I_5, I_3, I_1 as "basic" IPs and I_8, I_6, I_4, I_2 as intermediate ("compromise") ones.

To achieve this goal, the following tasks should be solved:

- to construct and solve the PCM of the IP;
- to justify a more acceptable iteration;
- to outline ways of further development of the PCM construction technology.

ESTABLISHING NORMALLY WEIGHTED COEFFICIENTS FOR SAATI PRIORITY INDEXES

So, the ordering of the studied IPs Saaty's scale is obvious:

$$IP_9 \succ IP_8 \succ IP_7 \succ IP_6 \succ IP_5 \succ IP_4 \succ IP_3 \succ IP_2 \succ IP_1, \qquad (10)$$

which greatly facilitates the application of MoP.

Let us further break down the ranking (10) into pairwise comparisons of the studied IPs:

$$IP_9 \succ IP_8 \quad IP_9 \succ IP_7 \quad IP_9 \succ IP_6 \quad IP_9 \succ IP_5$$
$$IP_9 \succ IP_4 \quad IP_9 \succ IP_3 \quad IP_9 \succ IP_2 \quad IP_9 \succ IP_1$$

$$IP_8 \succ IP_7 \quad IP_8 \succ IP_6 \quad IP_8 \succ IP_5 \quad IP_8 \succ IP_4$$
$$IP_8 \succ IP_3 \quad IP_8 \succ IP_2 \quad IP_8 \succ IP_1$$

$$IP_7 \succ IP_6 \quad IP_7 \succ IP_5 \quad IP_7 \succ IP_4 \quad IP_7 \succ IP_3$$
$$IP_7 \succ IP_2 \quad IP_7 \succ IP_1$$

$$IP_6 \succ IP_5 \quad IP_6 \succ IP_4 \quad IP_6 \succ IP_3 \quad IP_6 \succ IP_2$$
$$IP_6 \succ IP_1$$

$$IP_5 \succ IP_4 \quad IP_5 \succ IP_3 \quad IP_5 \succ IP_2 \quad IP_5 \succ IP_1$$

$$IP_4 \succ IP_3 \quad IP_4 \succ IP_2 \quad IP_4 \succ IP_1$$

$$IP_3 \succ IP_2 \quad IP_3 \succ IP_1$$

$$IP_2 \succ IP_1 .$$

Next, we construct a square matrix of IP $C=\|c_{jk}\|$ adjacency (**Table 2**), giving the following quantitative estimates to the indicators of preference:

$$c_{ij} = \begin{cases} \forall j \ne k: & \begin{cases} 2, & if \quad IP_j \succ IP_k \\ 0, & if \quad IP_k \succ IP_j \end{cases} \\ \forall j = k: & 1 \end{cases} . \qquad (11)$$

As follows from the results of the first iteration of the MoP, they are linear and therefore unacceptable. Therefore, the application of the MoP was continued (**Table 3**).

It is easy to see that at each subsequent iteration, the quantitative assessment of the distance between the studied linguistic IPs becomes more and more differentiated, and the values of the established NWCs β_{IP_k} become more and more nonlinear (**Table 3**).

From **Table 3** we also see that the results of the I and V iterations of the MoP are clearly unacceptable. After all, in the I iteration, as already noted, linear NWCs were obtained, and the V iteration went from the accepted accuracy of calculations to the fourth decimal place to a higher level. As a result $\beta_{I_k}^{V} = 0$, which is also unacceptable. **Figure 2** gives a visual representation of the dynamics of differentiation of the sought NWCs of the IP depending on the MoP iteration.

Analyzing in detail the contents of **Table 3** and **Figure 2**, we will consider the results of the II iteration of the MoP to be more acceptable for research purposes. Since, firstly, they are nonlinear; secondly, they are suitable for the accuracy of the calculations of the NWC.

Further, in the Microsoft Excel software environment, the results of the second iteration of the MoP were described by the following step function

$$\beta_{I_k} = 0.0021 \cdot x\left(I_k\right)^{2.2664}, \qquad (12)$$

which opened up prospects for further analytical development of the technology of applying AHR.

Table 2

Square matrix of incidences (pairwise comparisons) of linguistic priority levels of the Saaty scale

IP_k	IP_9	IP_8	IP_7	IP_6	IP_5	IP_4	IP_3	IP_2	IP_1	Σ	β	β*
1	2	3	4	5	6	7	8	9	10	11	12	13
IP_9	1	2	2	2	2	2	2	2	2	17	0,209877	0,2099
IP_8	0	1	2	2	2	2	2	2	2	15	0,185185	0,1852
IP_7	0	0	1	2	2	2	2	2	2	13	0,160494	0,1605
IP_6	0	0	0	1	2	2	2	2	2	11	0,135802	0,1358
IP_5	0	0	0	0	1	2	2	2	2	9	0,111111	0,1111
IP_4	0	0	0	0	0	1	2	2	2	7	0,08642	0,0864
IP_3	0	0	0	0	0	0	1	2	2	5	0,061728	0,0617
IP_2	0	0	0	0	0	0	0	1	2	3	0,037037	0,0370
IP_1	0	0	0	0	0	0	0	0	1	1	0,012346	0,0123
									Σ	81	1	0,9999

Table 3

Results of successive iterations of the prioritization method

I_k	Iterations of the prioritization method				
	I	II	III	IV	V
1	2	3	4	5	6
IP_9	0,2099	0,2965	0,3717	0,4364	0,4921
IP_8	0,1852	0,2311	0,2566	0,2680	0,2703
IP_7	0,1605	0,1738	0,1682	0,1542	0,1375
IP_6	0,1358	0,1247	0,1031	0,0815	0,0633
IP_5	0,1111	0,0838	0,0576	0,0384	0,0256
IP_4	0,0864	0,0511	0,0281	0,0154	0,0087
IP_3	0,0617	0,0266	0,0112	0,0049	0,0019
IP_2	0,037	0,0102	0,0031	0,0011	0,0004
IP_1	0,0123	0,0020	0,0004	0,0001	0
Σ	1	1	1	1	1

The proposed quantitative scale (column 3 of **Table 3**) contributes to much greater sensitivity of priority measurements:

$$max\left(\frac{\beta_{I_k}}{\beta_{I_j}}\right) = \frac{max\ \beta_{I_k}}{min\ \beta_{I_j}} = \frac{\beta_{I_9}}{\beta_{I_1}} = \frac{0.2965}{0.0020} = 148.25. \quad (13)$$

Which is 16.5 (!) times better than the result (7) obtained for linear LP_{IP_k} estimates.

Moreover, the quantitative estimates of the ratio of linguistic IPs associated with the modifier "very" became noticeably more differentiated:

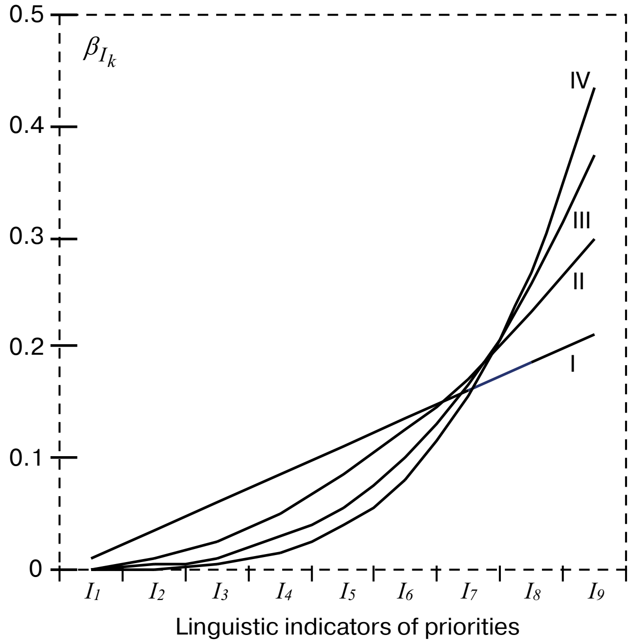

Fig. 2. The influence of iterations of the prioritization method on the nonlinearity and accuracy of the normalized weight coefficients of priority indicators: I–IV — method iteration numbers

$$\frac{\beta_{I_7}}{\beta_{I_6}} = \frac{0.1738}{0.1247} = 1.40\,, \quad (14)$$

which is 20 % better than result (5);

$$\begin{cases} \dfrac{\beta_{I_8}}{\beta_{I_7}} = \dfrac{0.2311}{0.1738} = 1.33 \\ \dfrac{\beta_{I_8}}{\beta_{I_6}} = \dfrac{0.2311}{0.1247} = 1.85 \end{cases}, \quad (15)$$

which is 17 % and 39 % better than the result (6), respectively.

Thus, the effectiveness of introducing nonlinear NWCs for the quantitative assessment of linguistic IPs Saaty's scale will be considered proven.

The next step of the research should be the application of these coefficients to construct PCMs of known MCDMPs and their solution.

CONCLUSIONS

Summarizing the new scientific results obtained and presented in this publication on improving the technology of constructing PCMs, and therefore AHP as a whole, we will draw attention to the following more important provisions.

1. The shortcomings of linear estimates of LP_{IP_k} linguistic IPs Saaty's scale have been proven, which, despite the numerous positive results of the use of AHP, does not allow us to consider them optimal.

2. The Saati IP scale was considered as a holistic 9-point scale with an obvious ranking of its linguistic IP_k components. This made it possible to construct the PCM and apply the MoP to establish nonlinear NWCs of linguistic IPs.

3. It was determined that the results of the second iteration of the MoP, which, on the one hand, are nonlinear, and on the other hand, are more acceptable for research purposes, are the results of the NWC calculations.

4. In the Microsoft Excel software environment, the results of the second iteration of the MoP were described by a step function, which opened up prospects for further analytical development of the AHP application technology.

5. It was found that nonlinearity increased the sensitivity of Saaty's scale to the priority measurement by 16,5 times. Quantitative estimates of the ratio of linguistic IPs associated with the modifier "very" improved in relation to the indicators established in the linear scale: by 20 % for the ratio IP $IP_7 \rightarrow IP_6$, and by 17 % and 39 % for the ratio $IP_8 \rightarrow IP_7$ and, $IP_8 \rightarrow IP_6$ respectively.

6. Thus, a generalized positive conclusion can be made regarding the solution of the tasks of this publication, and therefore the achievement of its goal.

Further research on the development of the theory and practice of the application of AHP should be carried out in the direction of verifying the effectiveness of the results obtained, β_{IP_k} i.e., by constructing the corresponding PCMs and resolving known MCDMPs.

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УТОЧНЕННЯ НЕЛІНІЙНИХ КІЛЬКІСНИХ ОЦІНОК ІНДИКАТОРІВ ПЕРЕВАГ ШКАЛИ СААТІ

Резюме. *Будь-який ефективний менеджмент супроводжується розв'язанням багатокритеріальних (БК) задач прийняття рішень (ПР). Серед відповідних методів і технологій вже декілька десятиліть провідне місце за ефективністю займає метод аналізу ієрархій (МАІ), розроблений Т. Л. Сааті. Реалізація МАІ полягає у визначенні локальних векторів пріоритетів — нормованих вагових коефіцієнтів (НВК) — на всіх рівнях ієрархії вирішуваної БК задачі ПР (ЗПР). Їхній подальший синтез призводить до глобального вектора – інтегративних оцінок досліджуваних альтернатив, яким (і лише яким) притаманна системна властивість емерджентності, що полегшує ґрунтовний вибір кращої з них.*

Попри позитивні результати чисельних досліджень щодо застосування та вдосконалення МАІ, незмінною залишається потреба побудови матриці попарних порівнянь (МПП), розв'язання якої, власне, і забезпечує отримання шуканих пріоритетів — НВК. Зазначена МПП формується на основі прямих та обернених лінійних бальних оцінок — кількісних еквівалентів лінгвістичних індикаторів переваг (ІП), що фактично є штучною підміною вимірів у бальній шкалі ранжирування вимірами в інтервальній. Відповідно, такі рішення не є оптимальними, адже лінійність не є властивістю людського мислення, а орієнтація на неї може призвести до хибних результатів.

Ранжирування лінгвістичних ІП шкали Сааті є очевидним, тому математичним методом розстановки пріоритетів (МРП), відомим у системному аналізі як «задача про лідера», було отримано нелінійні НВК усіх дев'яти ІП, виміряних уже в унікальній за кваліметричними властивостями абсолютній шкалі. Обґрунтовано прийнятність для досліджень результатів ІІ ітерації МРП. Аналітичний опис зазначеної нелінійності здійснено в ступеневій шкалі. Встановлено, що власне нелінійність збільшила чутливість шкали до виміру пріоритетності в 16,5 раза. Кількісні оцінки співвідношення лінгвістичних ІП, пов'язаних із модифікатором *"дуже", покращилися в порівнянні з показниками лінійної шкали: на 20 % – для співвідношення IP7–IP6, і відповідно на 17 % і 39 % – для співвідношення IP8–IP7 і IP8–IP6. Це свідчить про вищий рівень гармонізації кількісно-якісної адекватності шкали Сааті.*

Ключові слова: *прийняття рішень, метод аналізу ієрархій, шкала Сааті, нелінійна дефазифікація лінгвістичних індикаторів пріоритетів, нормалізовані вагові коефіцієнти.*

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